



ORIGINAL ARTICLE

Analysis of buoyancy assisting and opposing flows of colloidal mixture of titanium oxide, silver, and aluminium oxide nanoparticles with water due to exponentially stretchable surface



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Received 8 October 2022; accepted 4 January 2023

Available online 10 January 2023

KEYWORDS

Ternary nanofluid;
Thermal radiation;

Abstract Energy is essential for a nation's economic growth. Energy is recognized in contemporary society as being crucial to the development of quality of life and sustainability. The environment transforms/absorbs heat and sunlight in a variety of ways. Some of these transitions lead

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Peer review under responsibility of King Saud University.



Nomenclature

Symbols

T	Temperature (K)
T_w	Wall temperature (K)
T_∞	Far-field temperature (K)
T_0	Reference temperature (K)
q_r	Thermal radiation heat flux
k^*	Mean absorption coefficient (m^{-1})
K^*	Permeability of porous medium (m^2)
K_1	Porous permeability parameter
U	Velocity (ms^{-1})
Nu_x	Nusselt number
f	Dimensionless velocity profile
l	length (m)
k	Thermal Conductivity ($kgms^{-3}K^{-1}$)
C_p	Specific Heat ($Jkg^{-1}K^{-1}$)
Pr	Prandtl number
Rd	Radiation parameter
u and v	Velocity components (ms^{-1})
x and y	Coordinates (m)
Cf_x	Skin friction
Re	Local Reynolds number

Greek Symbols

Ψ	Stream function
β	Thermal expansion (K^{-1})
ν	Kinematic Viscosity (m^2s^{-1})
σ^*	Stefan-Boltzmann constant ($Kgs^{-3}K^{-4}$)
μ	Dynamic Viscosity ($kgm^{-1}s^{-1}$)
λ	Mixed convection parameter
θ	Dimensionless temperature profile
η	Similarity variable
Λ	Solid volume fraction
ρ	Density (kgm^{-3})

Subscripts

mnf	Modified nanofluid
nf	Nanofluid
$S1, S2, S3$	Solid particles
hnf	Hybrid nanofluid
f	Fluid

Exponentially stretching sheet;
Buoyancy assisting and opposing flow

to the flow of renewable energy sources like wind and biomass. Solar energy has become one of the promising alternative energy sources in the future because to the improvements made to enhance its performance. In this context, the impact of solar radiation on modified nanofluid flow over an exponential stretching sheet is examined. Using the proper similarity transformations, the governing equations for the flow assumptions are reduced to ordinary differential equations. The numerical simulation of these simplified equations is then performed using the Runge-Kutta Fehlberg method and the shooting methodology. With the aid of graphs and tables, the effects of numerous parameters on the involved fields are described. Results reveal that the modified nano liquid shows increased heat transport for opposing flow situation than the assisting flow situation for incremented values of porosity parameter and volume fraction. The modified nanoliquid shows increased heat transport for opposing flow situation with respect to augmented values of radiation parameter.

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1. Introduction

A nanofluid (NF) is a fluid that contains nanosized particles in addition to the base fluid. Colloidal suspensions of nanosized particles in a base fluid yields NFs. The thermal performance of NFs is predicted to be better than that of conventional fluids due to the suspended nanoparticles (NPs) presence high thermal conductivity. In this reference frame, Choi and Eastman (Choi and Eastman, 1995) investigated the usage of NPs to advance the thermal conductivity of liquids in 1995. The various applications of nanofluid involve solar energy (Mahian et al., 2013), evaporating system and condensing system (Rashidi et al., 2018; Ezaier et al., 2022; Ezaier et al., 2022), and heat transfer devices (Sajid and Ali, 2019; Ezaier, 2022). Recently, numerous researchers discussed the flow, heat and mass transport behaviour of diverse liquid streams by considering different NPs suspension under the impact of several influencing factors over different geometries (Saleem et al., 2022; Anantha Kumar et al., 2022; Sandeep et al., 2022; Zaydan et al., 2020; Awan et al., 2020; Ashraf et al., 2022). In this era of rapidly developing technology, a new subclass

of nanofluids known as hybrid NFs has been produced. This subclass is a modified version of the original nanofluids. In contrast to traditional NFs, hybrid NFs are made up of more than one metallic nanoparticle. Recently, a number of researchers have discussed the flow, heat and mass transport behavior of various liquid streams by considering dual NPs suspension under the influence of a number of different influencing factors over a variety of geometries (Cao et al., 2022; Shah et al., 2022; Song et al., 2021; Saleh, 2022; Gowda et al., 2021; Ashwinkumar et al., 2022; Awan et al., 2022). These researchers considered the impact of several influencing factors on the flow of the dual NPs suspension.

Nowadays, researchers are taking steps to add different NPs to the hybrid NF mixture and the resulting mixture is known as a modified NF. Two-particle NFs were used in hybrid NF research in the beginning, but more recently, a new class of functioning liquids has been developed and is being intensively studied. Three dissimilar types of nanoparticles were suspended in a base fluid subsequently by scientists and academics in order to create a new class of nanofluid. A ternary hybrid nanofluid (THNFs) is one of the innovative types of nanofluids.

Researchers were drawn to change the existing nano liquid as a result of the amplified need for cooling agents combined with high thermal capability at the industrial level, which led to the outline of trihybrid nanofluid with upgraded thermal properties. With this reason more experimental studies have been carried out to advance the thermal properties of an existing hybrid nanofluid by suspending three dissimilar types of solid nanoparticles in carrier liquid to form Trihybrid nanofluids. Recently, Yogeeshha et al. (Yogeeshha et al.) investigated the physical effects of blowing, Soret, and Dufour on an unstable surface occupied in a permeable medium containing a THNF. The impact that the various dimensionless limitations have had on the important profiles is shown pictorially here. Abbas et al. (Abbas et al., 2021) swotted the transport of a modified NF flow with time-dependent viscosity on an exponential Riga plate. The primary objective of this study was to investigate the heat transport efficiency that gains greater in comparison to HNF and NF systems. Xiu et al. (Xiu et al., 2022) swotted the forced convection stream of THNFs due to dual stretching on wedge surfaces when NPs volume is large and small. In this scenario, the researchers came to the conclusion that as time went on, in all of the four cases of water at various temperatures as well as in the case of four THMF flows, the same thing happened. Kumar et al. (Naveen Kumar et al., 2022) conferred the water based THNF flow over a surface with magnetic dipole effect. The influence that the various dimensionless limitations have had on the important profiles is shown here. Elnaqeeb et al. (Elnaqeeb et al., 2021) swotted the three-dimensional flow of THNF with water conveying NPs by considering suction and dual-stretching. In this particular instance, they came to the conclusion that an increase in suction and stretching ratio led to an increase in the vertical velocity of the motion along the x-direction, as well as an increase in the Nusselt number.

A porous media can be considered as a solid structure containing passages through which fluids can flow. Porous media has numerous applications in engineering, agriculture, building, and geothermal sciences. Lately, energy conversion and energy storage are two key ongoing domains of application for porous materials, where porous materials are necessary for supercapacitors, fuel cells, and batteries. Xia (Xia et al., 2021) explored the stream and heat transport in a Taylor-Couette streams of second-grade liquids subject to Lorentz and Darcy-Forchheimer quadratic drag forces. Rasool et al. (Rasool et al., 2022) investigated the influence of radiation and magnetic field on convection flow across a riga pattern with porous media. A comparative investigation of the LTNE condition for NF flow through porous media was conducted by Prasannakumara (Prasannakumara, 2022). Gowda et al. (Punith Gowda et al., 2021) investigated the impact of radiation and heat production on second grade fluid flow in a permeable medium. Madhukesh et al. (Madhukesh et al., 2021) investigated the impact of particle deposition on Casson hybrid NF flow in a permeable medium.

Thermal radiation is one of the three main kinds of heat transport and it is a basic feature of matter that depends on its temperature. From a microcosmic perspective, thermal radiation is caused by random particle motion (caused by the material's temperature), which accelerates the charge or induces the electrons and protons to oscillate in a dipole. Wakif (Wakif, 2020) studied the dynamics of thermal conductivity and temperature-dependent viscosity on steady convective flows of radiative Casson liquids over a sheet. Wakif et al. (Wakif et al., 2021) explored how heat radiation affected Walters-b fluid flow across a surface due to mixed convection. The effects of thermal radiation on flow of a NF over a surface was studied by Ganesh et al. (Vishnu Ganesh et al., 2018). Wang et al. (Wang, 2022) investigated the effect of thermal radiation on heat transport in stream of NF. Alhadhrami et al. (Alhadhrami, 2021) investigated the effect of thermal radiation on heat transport in plane wall jet stream of Casson NF with suction.

Buoyancy forces occur as a result of density fluctuations in a fluid subject to gravity, and they create a wide range of important phenomena in many fields of fluid mechanics. Progress in this field has generally been driven by a desire to find solutions to very real issues, like

those in hydraulic engineering or meteorology sectors. Khan et al. (Khan et al., 2021) swotted the buoyant effect on the flow of hybrid NPs along a vertical plate using the Darcy Brinkman model. Guderu et al. (Mohammed Alshehri et al., 2021) imposed the Cattaneo Christov model to investigate the thermal transfer rate of an SWCNT/MWCNT nanofluid including micropolar fluid via a partly slipped vertical riga plate. Sandeep et al. (Sandeep et al., 2022) examined the convective phenomenon with a buoyancy effect in a nanofluid flow. Samrat et al. (Samrat et al., 2022) analyzed the radiation effects of the stream of an unstable non-Newtonian fluid. Li et al. (Li et al., 2022) swotted the flow of a Maxwell micropolar hybrid NF in an exponentially stretched/shrunk vertical sheet approaching a stagnation point comprising buoyant assisting and opposing flow.

It is not always necessary for the stretching surface to be linear, as in situations involving stretching plastic sheets. Due to its usage in the manufacture of plastic films, polymer sheets, metal wires, paper, glass fiber and other materials, the heat transfer analysis of boundary layer flow across an exponentially extending surface with specified temperature or heat flux has drawn a lot of interest. Neethu et al. (Neethu et al., 2022) conducted research using an exponentially extending sheet to investigate multiple linear regression on a bioconvective hybrid NF flow. Guderu et al. (Guedri et al., 2022) investigated the three-dimensional flow and thermal transport of a third-grade fluid across an exponentially stretched sheet. Awan et al. (Awan et al., 2020) numerically investigated the chemical effects on the second-grade fluid flow across an oscillatory stretched sheet. Krishnamurthy et al. (Krishnamurthy et al., 2015) explored the role of viscous dissipation and magnetic field on the flow and heat transport of a NF using an exponentially stretched sheet. Sandhya et al. (Sandhya et al., 2021) investigated the impact of buoyant forces and activation energy on a hydromagnetic radioactive flow along an exponentially stretched surface.

In view of the above-served literature, no work is carried out on an exponentially stretching sheet in assisting and opposing the behavior of ternary nanofluid including thermal radiation and porous medium. Here, we considered suspension of three different NPs in the base liquid water. According to the available research, one of the key aspects of channel flows is water dynamics. This applies regardless of the amount of the water. Water is commonly used in commercial settings due to the fact that it can be obtained without much trouble, is very inexpensive, and may be dissolved to produce a solution, suspension, or colloidal suspension. Because of this, the incorporation of nanoparticles is required in order to alter the attributes of water's poor thermal conductivity, which is caused by the fact that water naturally has a pH of 7 and may exist in three different states while maintaining a high surface tension (Animasaun et al., 2022).

Based on the aforementioned accessible literature analysis. No work is conducted to study the impact of assisting and opposing flow behavior with porous and thermal radiation in the combination of ternary nanofluid over an exponential stretched surface. The present investigation is carried out to investigate assisting and opposing the behavior of ternary nanofluid over an exponential stretched surface including thermal radiation and porous medium. The governing equations that represent momentum and temperature equations are reduced to ODEs using similarity constraints. The results are numerically tackled with RKF-45 and the shooting procedure. The analysis of the results and important engineering coefficients are discussed in brief. Finally, the purpose of this article was to answer the relevant research questions:

- What effect do several significant factors have on the dynamics of flow, mass, and heat transfer behavior?
- What is the impact of thermal radiation factor on temperature profile.
- What effect do several significant factors have on both assisting and opposing flows?
- What impact does increase of different parameters have on skin friction and heat transfer rate?

Table 1 The material characteristics of base liquid and NPs (see (Rosseland, 1931; Madhukesh et al., 2022; Rana and Bhargava, 2011)).

Material	$\rho(kg/m^3)$	$C_p(Jkg^{-1}K^{-1})$	$k(kgms^{-3}K^{-1})$	$\beta \times 10^{-5}(K^{-1})$
Aluminium oxide (Al_2O_3)	3970	765	40	0.85
Silver (Ag)	6500	540	18	1.89
Titanium Oxide (TiO_2)	4250	686.2	8.9538	0.9
Water (H_2O)	997.1	4179	0.613	21

Table 2 Validation with $[-\theta'(0)]$ values for some reduced cases.

Pr	(Bidin and Nazar)	(Abd El-Aziz, 2009)	(Magyari and Keller, 1999)	(Mukhopadhyay and Gorla, 2012)	(Ishak, 2011)	Present study
1	0.9547	0.954785	0.954782	0.9547	0.9548	0.954955
2	1.4714			1.4714	1.4715	1.4714207
3	1.8691	1.869074	1.869075	1.8691	1.8691	1.8690440
5		2.500132	2.500135	2.5001	2.5001	2.5001089
10		3.660372	3.660379	3.6603	3.6604	3.6603543

Table 3 Validation with $[f''(0)]$ values for some reduced cases.

(Khan et al., 2011) (VIPM) $f''(0)$	(Kameswaran et al., 2012) (RKF) $f''(0)$	Present study (RKF-45) $f''(0)$
-1.28138	-1.281809	-1.281805

$$\rho_{mf} = (1 - (\Lambda_1 + \Lambda_2 + \Lambda_3))\rho_f + \Lambda_1\rho_{S1} + \Lambda_2\rho_{S2} + \Lambda_3\rho_{S3} \quad (10e)$$

$$(\rho C_p)_{mf} = (1 - (\Lambda_1 + \Lambda_2 + \Lambda_3))(\rho C_p)_f + \Lambda_1(\rho C_p)_{S1} + \Lambda_2(\rho C_p)_{S2} + \Lambda_3(\rho C_p)_{S3} \quad (10f)$$

$$\frac{k_{mf}}{k_f} = \frac{\Lambda_1 k_1 + \Lambda_2 k_2 + \Lambda_3 k_3 + 2(\Lambda_1 + \Lambda_2 + \Lambda_3)k_f + 2(\Lambda_1 + \Lambda_2 + \Lambda_3)}{\Lambda_1 k_1 + \Lambda_2 k_2 + \Lambda_3 k_3 + 2(\Lambda_1 + \Lambda_2 + \Lambda_3)k_f - (\Lambda_1 + \Lambda_2 + \Lambda_3)} \quad (10g)$$

$$(\rho\beta)_{mf} = (1 - (\Lambda_1 + \Lambda_2 + \Lambda_3))(\rho\beta)_f + \Lambda_1(\rho\beta)_{S1} + \Lambda_2(\rho\beta)_{S2} + \Lambda_3(\rho\beta)_{S3} \quad (10h)$$

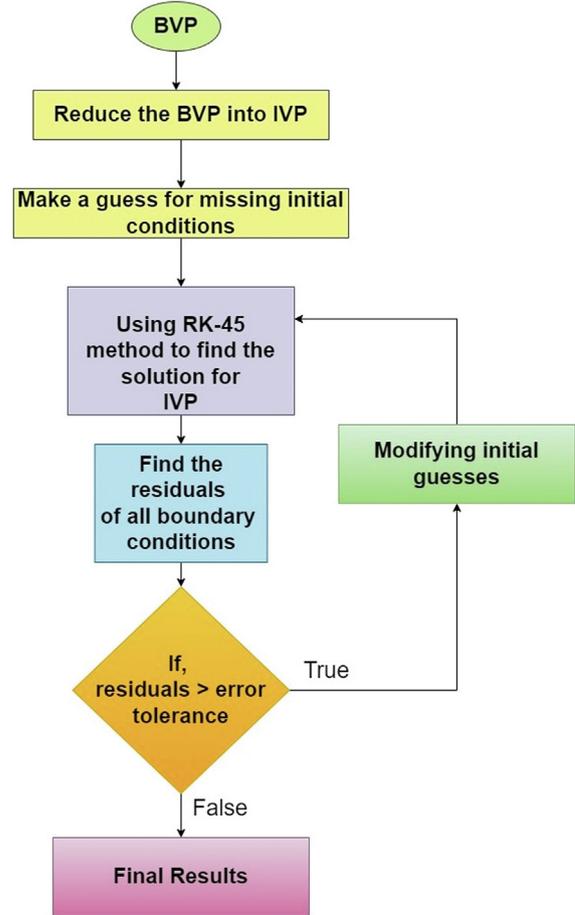
$$\mu_{mf} = \frac{1}{\mu_f^{-1}(1 - (\Lambda_1 + \Lambda_2))^{2.5}} \quad (10i)$$

$$\rho_{mf} = (1 - (\Lambda_1 + \Lambda_2))\rho_f + \Lambda_1\rho_{S1} + \Lambda_2\rho_{S2} \quad (10j)$$

$$(\rho C_p)_{mf} = (1 - (\Lambda_1 + \Lambda_2))(\rho C_p)_f + \Lambda_1(\rho C_p)_{S1} + \Lambda_2(\rho C_p)_{S2} \quad (10k)$$

$$(\rho\beta)_{mf} = (1 - (\Lambda_1 + \Lambda_2))(\rho\beta)_f + \Lambda_1(\rho\beta)_{S1} + \Lambda_2(\rho\beta)_{S2} \quad (10l)$$

$$\mu_{mf} = \frac{1}{\mu_f^{-1}(1 - (\Lambda_1))^{2.5}} \quad (10m)$$


Fig. 1b Numerical procedure algorithm.

$$\rho_{mf} = (1 - (\Lambda_1))\rho_f + \Lambda_1\rho_{S1} \quad (10n)$$

$$(\rho C_p)_{mf} = (1 - (\Lambda_1))(\rho C_p)_f + \Lambda_1(\rho C_p)_{S1} \quad (10o)$$

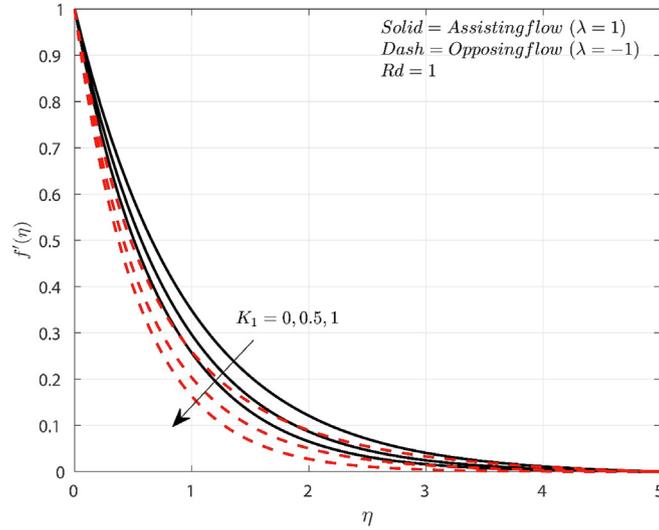


Fig. 2 Influence of K_1 on $f'(\eta)$.

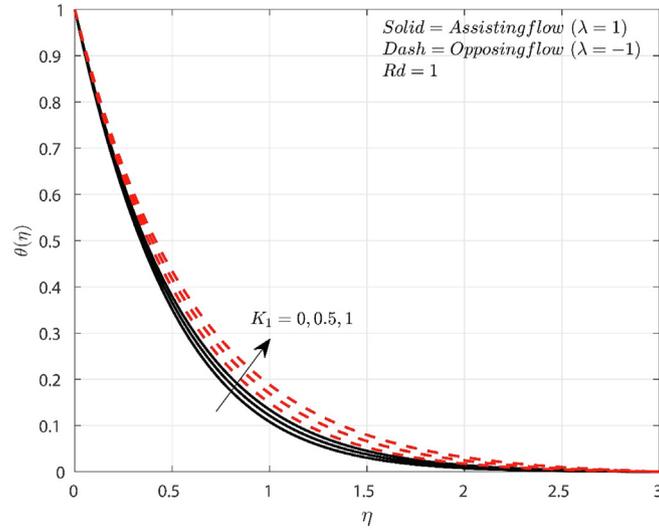


Fig. 3 Influence of K_1 on $\theta(\eta)$.

$$(\rho\beta)_{\eta f} = (1 - (\Lambda_1))(\rho\beta)_f + \Lambda_1(\rho\beta)_{S1} \quad (10p)$$

These equations (10d) to (10p) represent the thermophysical properties of the ternary hybrid nanofluid, hybrid nanofluid and nanofluid, respectively.

In the present study, the major engineering aspects like Cf_x and Nu_x are stated as (see (Madhukesh et al., 2022)). So, the expression for Cf_x (skin friction) is given as

$$Cf_x = \frac{2\tau_w}{\rho_f U^2} \quad (11)$$

The expression for Nu_x (Nusselt number) is given as

$$Nu_x = \frac{lq_w}{k_f(T_w - T_\infty)} \quad (12)$$

In the above expressions, the terms τ_w and q_w are stated as,

$$\begin{aligned} \tau_w &= \mu_{mf} \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad \text{and} \quad q_w \\ &= - \left(\frac{16\sigma^* T_\infty^3}{3k^*} + k_{mf} \right) \left(\frac{\partial T}{\partial y} \right)_{y=0} \end{aligned} \quad (13)$$

By using (13) in (11–12) the expression simplified to

$$Cf_x \sqrt{Re} = \frac{f''(0)}{A_1} \quad (14)$$

$$\frac{Nu_x}{\sqrt{Re}} = - \left(\frac{4}{3} Rd + \frac{k_{mf}}{k_f} \right) \theta'(0) \quad (15)$$

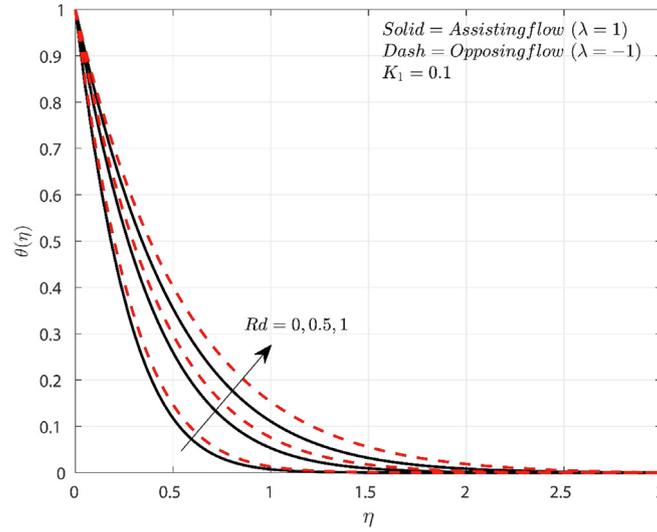


Fig. 4 Influence of Rd on $\theta(\eta)$.

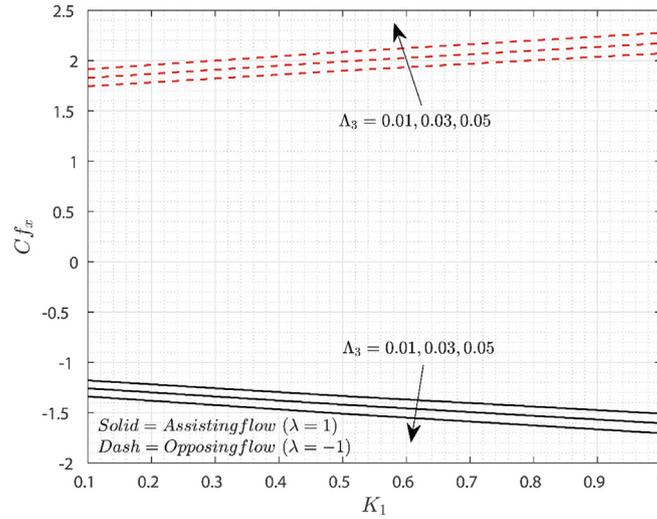


Fig. 5 Influence of Λ_3 and K_1 on Cf_x .

where $Re = \frac{U_\infty \bar{x}}{2\nu_f}$ is called the local Reynolds number.

3. Methodology and validation of the code

The RKF-45 strategy and shooting techniques (see refs. (Rehman et al., 2019; Ur Rehman et al., 2017; Rehman et al., 2017; Rehman et al., 2019; Rehman et al., 2017)) are utilised to solve the governing equations (7 and 8) and simplified boundary conditions using well-known computational tools. We convert the modified equations into a system of first-order.

To convert the system of equations into first order, let us choose $f = r_1, f' = r_2, f'' = r_3, \theta = r_4$ and $\theta' = r_5$. The equations become

$$r'_3 = -\left(A_1 A_2 (r_1 r_3 - 2(r_2)^2) - K_1 r_2 \pm \lambda A_1 A_4 r_4\right) \quad (16)$$

$$r'_5 = -A_3 \text{Pr} (r_5 r_1 - r_2 r_4) / \left(\frac{k_{mf}}{k_f} + \frac{4}{3} Rd \right) \quad (17)$$

And the boundary conditions are

$$r_1(0) = 0, r_2(0) = 1, r_3(0) = a_1, r_4(0) = 1, r_5(0) = a_2. \quad (18)$$

The IVP from equations (16 to 17) and (18) is quantitatively evaluated using the RKF-45 order approach, and unknown values are discovered using a shooting procedure with error tolerance and step size set to 10^{-6} and $h_1 = 0.01$ correspondingly, meeting the boundary conditions at the infinity level. The numerical estimates are derived by applying the MATLAB built-in program bvp4c by substituting the values of the parameters $K_1 = 0.1, Rd = 1 \& \lambda = \pm 1$ and utilizing the material characteristics of base liquid and NPs (see Table 1). The graphical results of each restriction are achieved by altering one parameter while holding the other parameters constant. The numerical validation of the code for $-\theta'(0) \& f''(0)$

values between the current study and the published research is shown in Table 2 and Table 3, and there is a fair level of agreement between both.

The algorithm of the RKF -45 is given below (see (Mathews and Fink, 2004):

$$\text{RK 4th order: } j_{i+1} = +\frac{25}{216}m_1 + \frac{2197}{4104}m_4 + \frac{1408}{2565}m_3 - \frac{1}{5}m_5 + j_i.$$

$$\text{RK 5th order: } k_{i+1} = \frac{2}{55}m_6 - \frac{9}{50}m_5 + \frac{16}{135}m_1 + \frac{28561}{56430}m_4 + \frac{6656}{12825}m_3 + j_i,$$

The six steps of the methodology as follows:

$$h_1f(n_i, j_i) = m_1,$$

$$h_1f\left(n_i + \frac{1}{4}h_1, j_i + \frac{1}{4}m_1\right) = m_2,$$

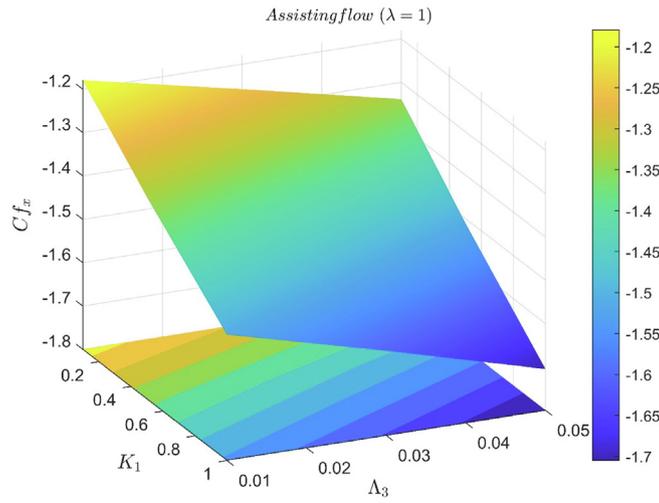
$$h_1f\left(n_i + \frac{3}{8}h_1, j_i + \frac{3}{32}m_1 + \frac{9}{32}m_2\right) = m_3,$$

$$h_1f\left(n_i + \frac{12}{13}h_1, j_i + \frac{1932}{2147}m_1 - \frac{7200}{2147}m_2 + \frac{7296}{2147}m_3\right) = m_4,$$

$$h_1f\left(n_i + h_1, j_i + \frac{439}{216}m_1 - 8m_2 + \frac{3680}{513}m_3 - \frac{845}{4104}m_4\right) = m_5,$$

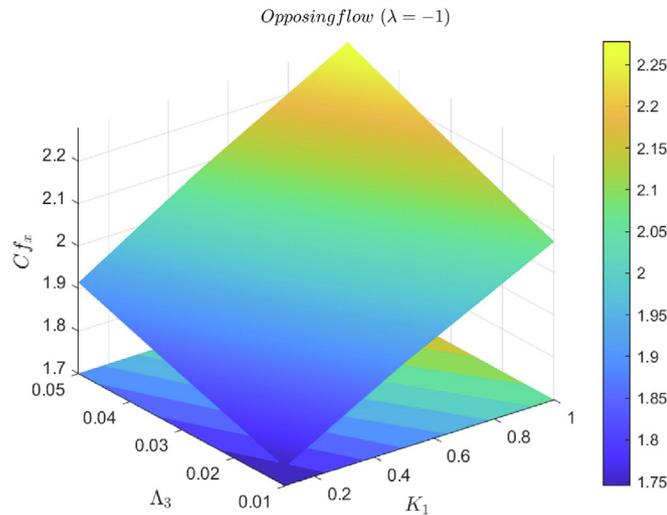
$$h_1f\left(n_i + \frac{1}{2}h_1, j_i + 2m_2 - \frac{8}{27}m_1 - \frac{11}{40}m_5 - \frac{3544}{2565}m_3 - \frac{1859}{4104}m_4\right) = m_6.$$

The flow chart of the numerical procedure is shown in the Fig. 1(b).



(a) Influence of Λ_3 and K_1 on Cf_x (assisting flow case).

Fig. 6a Influence of Λ_3 and K_1 on Cf_x (assisting flow case).



(b) Influence of Λ_3 and K_1 on Cf_x (opposing flow case).

Fig. 6b Influence of Λ_3 and K_1 on Cf_x (opposing flow case).

4. Results and discussion

The current work investigates the THNF flow over an exponential stretching sheet with porous medium. The modelling also takes thermal radiation into account. By utilizing a suitable collection of similarity variables, the simulated issue is reduced to a dimensionless form for the current analysis. Some significant factors were found throughout this procedure, and the impact of these relevant parameters on the involved profiles is shown graphically and briefly described. The graphs and tables illustrate the heat transfer rates and friction factors for various value of different parameters. In the figures, the solid lines indicate assisting flow condition whereas, the dashed lines indicate opposing flow condition.

The impact of K_1 on $f'(\eta)$ for both opposing and assisting flow cases is displayed in Fig. 2. Here, increase in K_1 value decreases the $f'(\eta)$ for both flow conditions. The resistance of the system is improved by raising values of the porosity component. The larger frictional force causes the fluid flow to decrease. The resistance of the surface to liquid mobility in this situation is enhanced by increasing porosity. The increased resistance reduces liquid velocity. Moreover, the $f'(\eta)$ declines faster for opposing flow case than the assisting flow case for incremented values of K_1 .

The effect of K_1 on $\theta(\eta)$ for both flow cases is displayed in Fig. 3. Here, upsurge in K_1 value increases the $\theta(\eta)$ for both flow conditions. It is clear that the presence of a porous medium reasons for an advanced restriction to fluid flow, which leads to a decline in fluid velocity and an increase in the thickness of the thermal boundary layer and enhances $\theta(\eta)$. Furthermore, the modified nanoliquid shows increased heat transport for opposing flow situation than the assisting flow situation for incremented values of K_1 .

The upshot of Rd on $\theta(\eta)$ for both flow cases is showed in Fig. 4. Here, upsurge in Rd value increases the $\theta(\eta)$ for both flow situations. The mean absorption coefficient is inversely related to Rd and decreases as Rd increases. Due to conductive heat transmission which is more effective than radiative heat transfer, the buoyancy force has decreased. In fact, a higher

Rd effectively transports more heat to operating fluids, increasing thermal dispersion. The fluid becomes more heated when the Rd is set to a high value. The heat transmission thereafter considerably rises. Moreover, the modified nanoliquid shows increased heat transport for opposing flow situation than the assisting flow situation for incremented values of Rd .

Impact of Λ_3 and K_1 on Cf_x for both flow cases is displayed in Fig. 5. Here, augmented values of Λ_3 and K_1 improve the Cf_x in opposing flow and reverse behaviour is observed for assisting flow situation. This is due to the fact that improvement in the porous parameter will opposes the motion of the liquid also improvement in the solid volume fraction will enriches the momentum boundary layer as a result surface drag force increases Moreover, the modified nanoliquid show improved friction factor for opposing flow case than the assisting case with respect to increased Λ_3 and K_1 values. Figs. 6(a) and 6(b) represents the three-dimensional representation of friction factor for both assisting and opposing flow conditions.

Impact of Λ_3 and Rd on Nu_x for both flow cases is displayed in Fig. 7. Here, improved values of Λ_3 advances the Nu_x for both flow situations but opposing trend is seen for increased Rd values. This is due to the fact that, increases in the values of k^* which leads to decrease Rd The radiative thermal flux indices and radiative heat transfer rates into the liquid will enhance as a consequence. The thickness of the thermal boundary layer is noticed due to improvement in Λ_3 and enhanced radiative thermal transmission. Moreover, the modified nanoliquid show improved heat transport rate for assisting flow case than the opposing case with respect to increased Λ_3 and Rd values. Figs. 8(a) and 8(b) represents the three-dimensional representation of heat transport rate for both assisting and opposing flow conditions. Computational values of $f''(0)$ and $\theta'(0)$ for dimensionless constraints is tabulated in Table 4 keeping $\Lambda_1 = \Lambda_2 = 0.01$. Here, the rise in values of Λ_3 and K_1 increases the $-f''(0)$ for both flow situations. Whereas, the Rd has negative impact on $-f''(0)$ for assisting flow case and positive impact $-f''(0)$ for opposing flow case. Finally, the rise in Λ_3 , Rd and K_1 declines the $-\theta'(0)$ for opposing flow case.

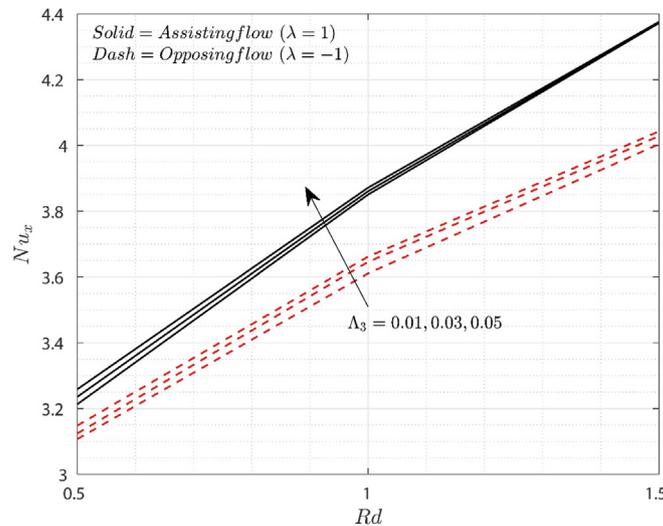
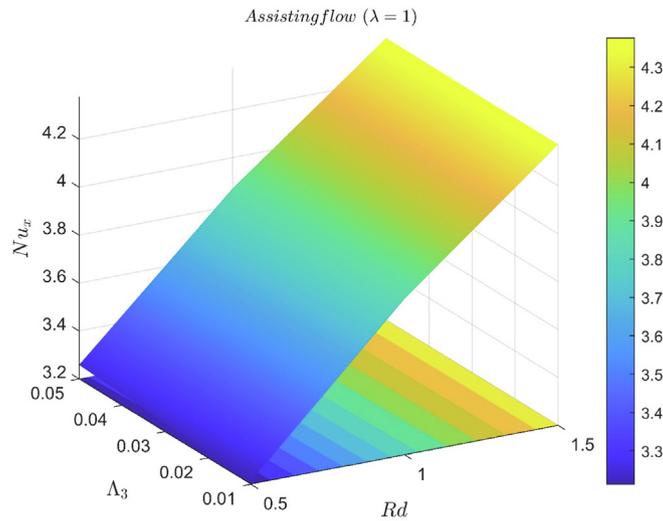
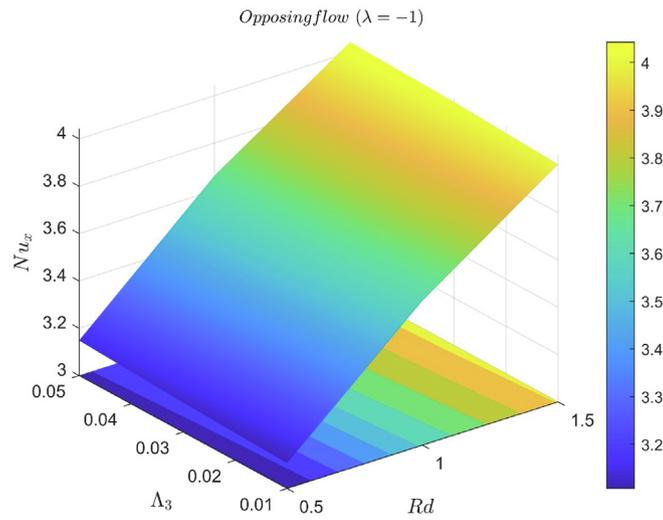


Fig. 7 Influence of Λ_3 and Rd on Nu_x .



(a) Influence of Λ_3 and Rd on Nu_x (assisting flow case).

Fig. 8a Influence of Λ_3 and Rd on Nu_x (assisting flow case).



(b) Influence of Λ_3 and Rd on Nu_x (opposing flow case).

Fig. 8b Influence of Λ_3 and Rd on Nu_x (opposing flow case).

Table 4 Computational values of $f''(0)$ and $\theta'(0)$ for dimensionless constraints keeping $\Lambda_1 = \Lambda_2 = 0.01$.

Parameters			Assisting flow $\lambda = 3$		Opposing flow $\lambda = -3$	
Λ_3	K_1	Rd	$-f''(0)$	$-\theta'(0)$	$-f''(0)$	$-\theta'(0)$
0.01	0.1	1	1.272510067	1.893916857	1.428568475	1.861824562
0.03			1.272510070	1.893916868	1.428568480	1.861824558
0.05			1.272510070	1.893916868	1.428568480	1.861824558
0.01	0.1	1	1.272510067	1.893916857	1.425161820	1.812303724
	0.5		1.412462391	1.858980580	1.564306516	1.775536556
	1.0		1.572160912	1.818845927	1.722411822	1.733515855
0.01	0.1	0.5	1.284222776	2.385390279	1.414105182	2.277281476
		1	1.272510022	1.893916483	1.425161760	1.812304283
		1.5	1.263480468	1.603316161	1.433959327	1.531010992

5. Final remarks

The current study will examine the effects of thermal radiation and the porous medium when modified nanofluid flow is present across an exponential stretching sheet. Additionally, the work investigates two distinct scenarios of modified nanofluid flow, namely (i) assisting flow case and (ii) opposing flow case. Using similarity variables, the set of governing equations are converted into ODEs. Furthermore, the RK4-5 and shooting methods are used to numerically solve these equations. Graphs are used to illustrate the significant dimensionless constraints on flow and thermal profiles. Using tables and graphs the significant engineering coefficients are described. The key points are the following as

- The $f'(\eta)$ declines faster for opposing flow case than the assisting flow case for incremented values of K_1 .
- The modified nanofluid shows increased heat transport for opposing flow situation than the assisting flow situation for advanced values of K_1 .
- The modified nanofluid shows increased heat transport for opposing flow situation than the assisting flow situation for augmented values of Rd .
- The modified nanofluid show improved friction factor for opposing flow case than the assisting case with respect to increased Λ_3 and K_1 values.
- The modified nanofluid show improved heat transport rate for opposing flow case than the assisting case with respect to increased Λ_3 and Rd values.

The current investigation is useful in the applications like supercapacitors, fuel cells, batteries, nanodevices and energy storage. The present work is conducted to investigate the ternary nanofluid flow across an exponential stretched surface with thermal radiation, porous medium and buoyancy effects. The current work can be extended to investigate concentration and bioconvection using different non-Newtonian fluids.

CRedit authorship contribution statement

J.K. Madhukesh: Conceptualization, Methodology, Software, Formal analysis, Validation, Writing – original draft. **R. Naveen Kumar:** Conceptualization, Methodology, Software, Formal analysis, Validation, Writing – original draft. **Umair Khan:** Conceptualization, Methodology, Software, Formal analysis, Validation, Writing – original draft. **Rana Gill:** Writing – original draft, Data curation, Investigation, Visualization, Validation. **Zehba Raizah:** Writing – original draft, Data curation, Investigation, Visualization, Validation. **Samia Elattar:** Conceptualization, Writing – original draft, Writing – review & editing, Supervision, Resources. **Sayed M Eldin:** Validation, Investigation, Writing – review & editing, Formal analysis. **S.H.A.M. Shah:** Provided significant feedback and assisted in the revised version of manuscript. Further, he also assisted us in revising the manuscript critically for important intellectual content. **B. Rajappa:** Writing – review & editing, Software, Data curation, Validation, Resources. **Ahmed M. Abed:** Writing – review & editing, Software, Data curation, Validation, Resources.

Acknowledgments

The author (Z. Raizah) extend her appreciation to the Deanship of Scientific Research at King Khalid University, Abha, Saudi Arabia, for funding this work through the Research Group Project under Grant Number (RGP.2/154/43). Also, this work was supported by the Princess Nourah bint Abdulrahman University Researchers Supporting Project number (PNURSP2023R163), Princess Nourah bint Abdulrahman University, Riyadh, Saudi Arabia. Moreover, this work is also supported via funding from Prince Sattam bin Abdulaziz University project number (PSAU/2023/R/1444).

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