



## ORIGINAL ARTICLE

# Natural convection in nanofluid flow with chemotaxis process over a vertically inclined heated surface



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**Abstract** The thermal energy transport analysis with chemotaxis in the free convective flow of viscous nanofluid over stretchable vertically inclined heated sheet is addressed in this article. The fluid forced and free convection motion is investigated and discussed with physical reasoning. The fluid also contains microorganism heavy-bottom species, and their chemotactic motion is studied. In the light of Buongiorno model, the impact of Brownian motion and thermophoresis slip mechanism on thermal conduction in the nanofluid is analyzed. The work is based on the similarity analysis of governing partial differential equations (PDEs) which lead to non-dimensional ordinary differential equations (ODEs). The solution of resulting flow and heat equations is computed via bvp4c technique. The outcomes are represented in graphical abstract. It is noted that free convective flow field increases near to the surface of sheet then it decays to free stream exponentially. Higher magnitude of thermophoretic force boost up the thermal energy transport in nanofluid flow. The Brownian

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motion enhances temperature profile and lower down the convection velocity. Chemotaxis motion of species in nanofluid is increasing function of bioconvective Peclet number.

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## 1. Introduction

Convection is the mode of heat transfer it produce due to the movement of molecules within fluids. Convection is subdivided into forced and free convection. In the force convection the fluid flow initiates / produce due to external sources such as pump or fan, whereas no external sources are involved to fluid flow in the free convection, the flow is produced due to variation of density. If the forces and momentum transport rate are dominated than free convection is neglectable, while if the buoyance forces are dominated than the force convection is neglected. The boundary layer flow in which natural and forced convection are substantial is known as mixed convection. The force and free convection combinedly used in electronic cooling and nuclear reactor technology. The combined effect of force and free convection was proposed by Tao (Tao, 1960) and Cheng (Cheng, 1977). Imura et al. (Imura et al., 1978) and Wang (Wang, 1982) experimentally study the heat transfer towards the isothermally heated plate with combined impact of force and free convection. Nakayama and Koyama (Nakayama and Koyama, 1987) studied the boundary layer fluid flow saturated porous with combined impact of force and natural convection using integral method. Chamkha and Khaled (Chamkha and Khaled, 2000) scrutinized the heat and mass transport analysis of natural convective flow from the porous semi-infinite plate with external magnetic field. Al-Mousawe et al. (Al-Mousawe et al., 2021) observed the combinedly free and forced convection on the boundary layer with porous medium. The related studies in the direction of forced and free convection (mixed convection) are found in Refs. (Ullah et al., 2021; Xia et al., 2022; Gupta et al., 2022). The present study also focuses on the investigation of the bioconvection phenomenon. The phenomenon occurs due to microorganism density gradient. In bioconvection the heavy bottom particles bonus upwards, when their density reduces due to upswimming speed and create balance between gravitational and viscous torque. Bioconvection has many applications in biological systems and biotechnology. The benefits of adding motile microorganisms to the suspension include enhanced mass transfer, micro scale mixing especially in micro volumes and improved nanofluid stability. A combination of nanofluids and bioconvection is consequently captivate for novel micro fluidic devices. The effect of bioconvection on fluid flow was first studied by Platt (Platt, 1961). The heat and mass transfer investigation of bioconvective nanofluid flow containing gyrotactic microorganisms across the stretching surface was manipulated by Kuznetsov (Kuznetsov, 2010; Kuznetsov, 2011). Uddin et al. (Uddin et al., 2016) discussed the gyrotactic bioconvection nanofluid flow saturated in porous media with multiple slip and Stefan blowing effects. Nadeem et al. (Nadeem et al., 2019) discussed the bioconvection micropolar nanofluid flow towards an exponentially stretching surface with microorganism. Khan et al. (Khan et al., 2020) investigated the bioconvection nanofluid flow between two stretchable rotating disks with entropy generation. Some recent investigation of the bioconvection nanofluid flow is presented in the Refs. (Khan et al., 2022; Habib et al., 2022; Wang et al., 2022; Gangadhar et al., 2022; Wang et al., 2022).

In industrial, metallurgical, manufacturing processes, and engineering, flow due to stretchable surface/sheet has a variety of applications. It is visible in the aerodynamic extrusion of plastic sheets and fibers, tinning, crystal growth, annealing of copper wire, and paper manufacture, glass blowing, and drawing. In the manufacturing of plastic and glass, the rate of cooling is significantly reliant on the feature of the finished product. Sakiadis (Sakiadis, 1961) was the first to investigate the flow of the boundary layer due to the continuously stretchable surface.

Crane (Crane, 1970) elaborated on this concept for exponentially stretching surfaces. Tsou et al. (Tsou et al., 1967) investigated the measurement of heat transfer over a boundary layer flow due to a stretched surface. Besthapu et al. (Besthapu et al., 2017) inspected the MHD thermally stratified mixed convection viscous nanofluid flow subject to viscous dissipation on an exponentially stretching surface. Ali et al. (Ali et al., 2020) analyzed the bioconvection micropolar base nanofluid flow with Cattaneo-Christov heat flux through a vertically stretching surface. Very recently, Yasir et al. (Yasir et al., 2022) studied the boundary layer flow due to unsteady cylindrical stretching surface. Khan et al. (Khan et al., 2022) discussed the heat source/sink effect in flow of magnetized Oldroyd-B fluid over a stretching surface. Much research recently attracts towards the study of stretching surface, which is found in the Refs. (Pattnaik et al., 2022; Alqarni et al., 2022; Wang, n.d.; Neethu et al., 2022; Wang, n.d.).

A significant role in regulating the product's quality is played by the topic of heat and mass transfer flows. Since they have a wide range of applications in many processes, including heat exchangers, crystal growth, the fiber industry hot rolling, etc., heat and mass transport phenomena over a stretching sheet or surface have recently been much study due to their significance. Analysis of generalized non-Newtonian fluid flow and thermal transportation in the presence of mixed nanostructures and dust particles was studied by Cheng et al. (Cheng et al., 2021). Abbasi et al. (Abbasi et al., 2022) analyzed heat transport analysis for blood flow driven by hybrid nanoparticles (Cu, Fe<sub>3</sub>O<sub>4</sub>) using a tapering complicated wavy curved channel. Hassan et al. (Waqas et al., 2022) discussed the effects of gyrotactic motile organisms on the flow of pseudoplastic nanofluid over a moving Riga plate with exponential heat flux. Bafakeeh et al. (Bafakeeh et al., 2022) examined the effects of Hall current and Soret effects on the unsteady MHD rotational flow of second-grade fluid via porous material under the presence of thermal radiation and chemical processes. The related similar studies can be found in Refs. (Liu et al., 2021; Manzoor et al., 2022; He and Abd Elazem, 2022; Shahid et al., 2022; Ahmed et al., 2022; Wang and Khater, 2023; Zuo, 2021).

Chemotaxis motion of microorganism species in viscous nanofluid free convective flow over heated inclined vertical sheet is the main concern of the present study. In the view of above comprehensive literature survey this research article is the novel contribution. This study shows the mechanism of both free and forced convective flow fields with the impact of various physical parameters. The whole mathematical analysis is performed with help of similarity transformation. The outcomes from governing differential equations are computed with help of bvp4c numerical technique. Each result is discussed with physical reasoning.

## 2. Formulation of flow and energy transport phenomena

Consider a heated elastic sheet which is immersed in infinite incompressible viscous nanofluid with microorganism species. The thermal and solutal energy equations are modeled with the effect of Brownian motion and thermophoresis. The effect of free and forced convection is also discussed in the current investigation. The sheet is vertical and rest in  $xy$ - plane under the influence of gravity force and makes an angle  $\alpha$  with  $y$ - axis and  $z$ - axis normal to the sheet. Suppose that sheet is stretched with velocity  $bx$  along  $x$ - axis, due to this the fluid motion occurs (see Fig. 1). Moreover, the sheet surface temper-

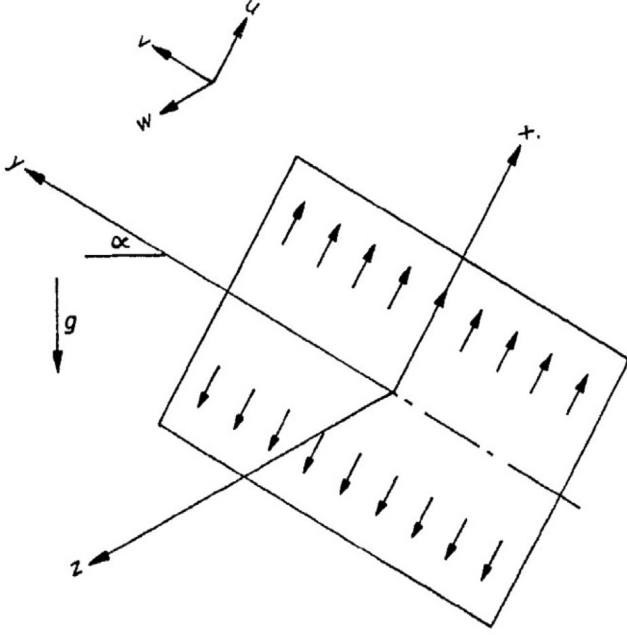


Fig. 1 Flow configuration.

ature is assumed  $T_w$  which is higher than free stream temperature  $T_\infty$ . Therefore, combine presence of thermal gradient and body force also produce the fluid motion. The velocity field is supposed as  $V = [u, v, w]$  along  $(x, y, z)$  - axes, respectively. Additionally, the nanoparticles and microorganism concentration at surface and free streams are  $(C_w, N_w)$  and  $(C_\infty, N_\infty)$ , respectively. All variable quantities are free not depends on  $y$  - axis, if we ignore the edge effects. The PDEs which governs the present problem are as follows:

Flow equations.

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = \nu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right] + g \cos \alpha B_T (T - T_\infty), \quad (2)$$

$$u \frac{\partial v}{\partial x} + w \frac{\partial v}{\partial z} = \nu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} \right] + g \sin \alpha B_T (T - T_\infty), \quad (3)$$

$$u \frac{\partial w}{\partial x} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right]. \quad (4)$$

Energy equations.

$$\left\{ \begin{aligned} u \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial z} - \tau \left[ D_B \left( \frac{\partial C}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial C}{\partial z} \frac{\partial T}{\partial z} \right) + \frac{\partial T}{\partial x} \left\{ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial z} \right)^2 \right\} \right] &= \alpha_1 \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right] \\ u \frac{\partial C}{\partial x} + w \frac{\partial C}{\partial z} &= D_B \left[ \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial z^2} \right] + \frac{\partial T}{\partial x} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right) \end{aligned} \right\} \quad (5)$$

Bioconvection equation.

$$u \frac{\partial N}{\partial x} + w \frac{\partial N}{\partial z} + \frac{b^* W_c}{C_w - C_\infty} \left[ \frac{\partial N}{\partial x} \frac{\partial C}{\partial x} + \frac{\partial N}{\partial z} \frac{\partial C}{\partial z} \right] + N \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial z^2} \right) = D_m \left( \frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial z^2} \right). \quad (6)$$

The appropriate boundary conditions for the flow and transport equations are Wang [45],

$$\begin{aligned} u = ax, v = 0, w = 0, T = T_w, C = C_w, N = N_w \text{ at } z \\ = 0, u \rightarrow 0, v \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty, N \rightarrow N_\infty \text{ as } z \\ \rightarrow \infty. \end{aligned} \quad (7)$$

In the above equations,  $\nu$  is the kinematic viscosity,  $\alpha_1 \left( = \frac{k}{\rho c_p} \right)$  the thermal diffusivity of nanofluid and  $(T, C, N)$  represent the temperature, concentration, and microorganism.

By using the flow ansatz proposed by Wang [45] as,

$$\begin{aligned} u = bx f'(\eta) + \Lambda \cos \alpha g(\eta), v = \Lambda \sin \alpha h(\eta), w = -\sqrt{av} f, \\ \eta = z \sqrt{\frac{a}{\nu}}, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \psi(\eta) \\ = \frac{N - N_\infty}{N_w - N_\infty}, \end{aligned} \quad (8)$$

the Eq. (1) is satisfied automatically and Eqs. (2–7) yield, (8).

$$f''' + ff'' - f^2 = 0, \quad (9)$$

$$g'' + fg' + \theta = 0, \quad (10)$$

$$h'' + fh' - hf' + \theta = 0, \quad (11)$$

$$\theta'' + Pr(f\theta' + N_b \theta' \varphi' + N_t \theta'^2 + Q_0 \theta) = 0, \quad (12)$$

$$\varphi'' + Le Pr f \varphi' + \frac{N_t}{N_b} \theta'' - K \varphi = 0, \quad (13)$$

$$h'' + L_b f h' + P_{be}(h' \varphi' + h \varphi'' + N_g \varphi''') = 0, \quad (14)$$

with boundary conditions.

$$\begin{aligned} f(0) = 0, f'(0) = 1, f'(\infty) = 0, g(0) = g(\infty) = 0, h(0) = h(\infty) \\ = 0 \end{aligned}$$

$$\theta(0) = \theta(\infty) = 0, \varphi(0) = \varphi(\infty) = 0. \quad (15)$$

The parameters contain in above equations are defined as  $Pr \left( = \frac{\nu}{\alpha_1} \right)$  the Prandtl number,  $N_b \left( = \frac{\tau D_B (C_w - C_\infty)}{\nu} \right)$  the Brownian diffusion coefficient,  $N_t \left( = \frac{\tau D_T (T_w - T_\infty)}{\nu T_\infty} \right)$  the thermophoresis parameter and  $Le \left( = \frac{\alpha_1}{D_B} \right)$  the Lewis number,  $P_{be} \left( = \frac{b W_c}{D_m} \right)$  the bioconvected Peclet number  $L_{be} \left( = \frac{\nu}{D_m} \right)$  the bioconvected Lewis number  $N_g \left( = \frac{N_\infty}{N_w - N_\infty} \right)$  the microorganism concentration difference factor.

### 2.1. Solution methodology

The non-linear system given in Eqs. (11 – 16) governs the present flow and energy transport problem due to free and forced convection is solved numerically through bvp4c Matlab builtin technique. This method based on the collocation and Richardson extrapolation schemes. For solution via this method the ODEs are transformed into first order by using the following variables:

$$f = y_1, f' = y_2, f'' = y_3, f''' = y_4', g = y_4, g' = y_5, g'' = y_5',$$

$$h = y_6, h' = y_7, h'' = y_7', \theta = y_8, \theta' = y_9, \theta'' = y_9',$$

$$\varphi = y_{10}, \varphi' = y_{11}, \varphi'' = y_{11}', \psi = y_{12}, \psi' = y_{13}, \psi'' = y_{13}'. \quad (16)$$

### 3. Results and discussion

The outcomes of present flow problem are acquired numerically and presented in the form of graphical abstracts. In this section a comprehensive discussion of these results is provided with physical justification. The dimensionless physical parameters are fixed as  $Pr = 4$ ,  $N_t = N_b = 0.5$ ,  $Le = 2$ ,  $Q = 0.5$

$Pe = 1.5$ ,  $L_b = 2.5$ . The graphs in Fig. 2(a, b) show the free convection velocity profile and temperature distribution in nanofluid for higher values of Prandtl number. It is observed that temperature field declines as increases as well as free convective velocity profile also. As Prandtl increases the thermal diffusion property of nanofluid declines in result the temperature profile lower down. Moreover, for large value of the thermal energy transportation in nanofluid occurs only near to surface of sheet. The impact of thermophoretic force due to nanoparticles on thermal profile and free convective veloc-

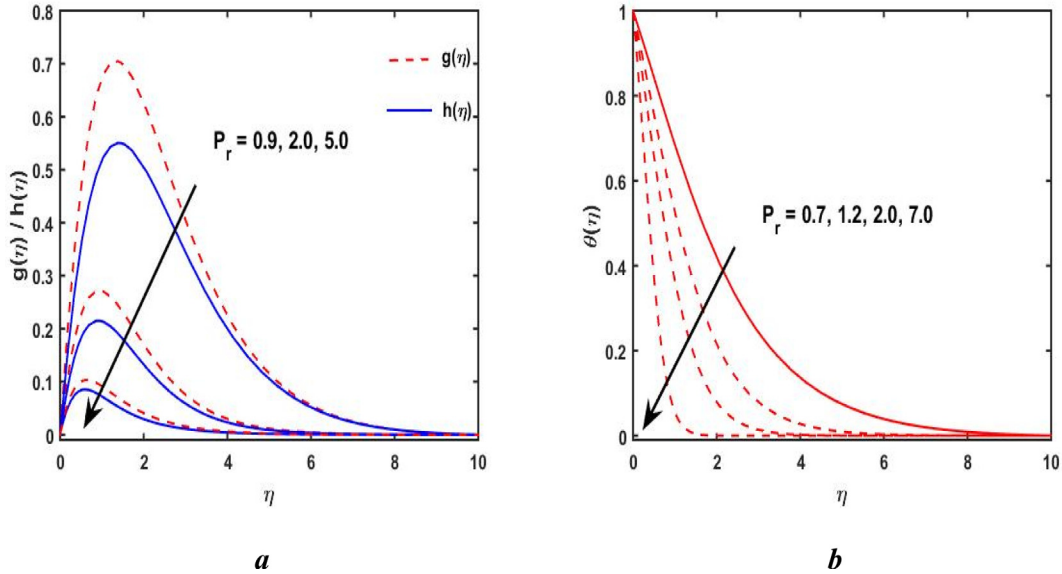


Fig. 2 (a, b): Free convective velocity profile and temperature distribution via  $Pr$ .

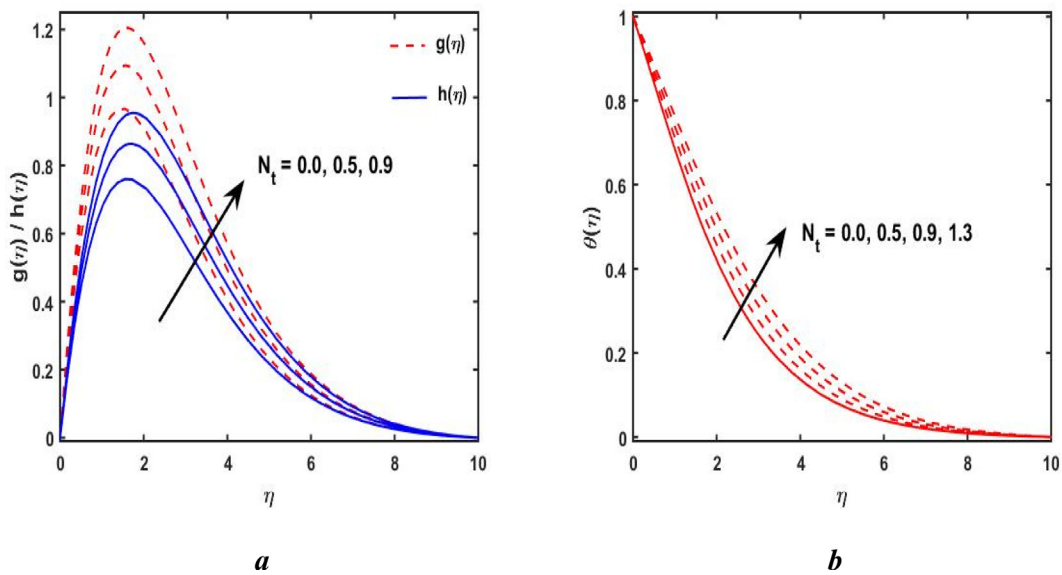


Fig. 3 (a, b): Free convective velocity profile and temperature distribution via  $N_t$ .

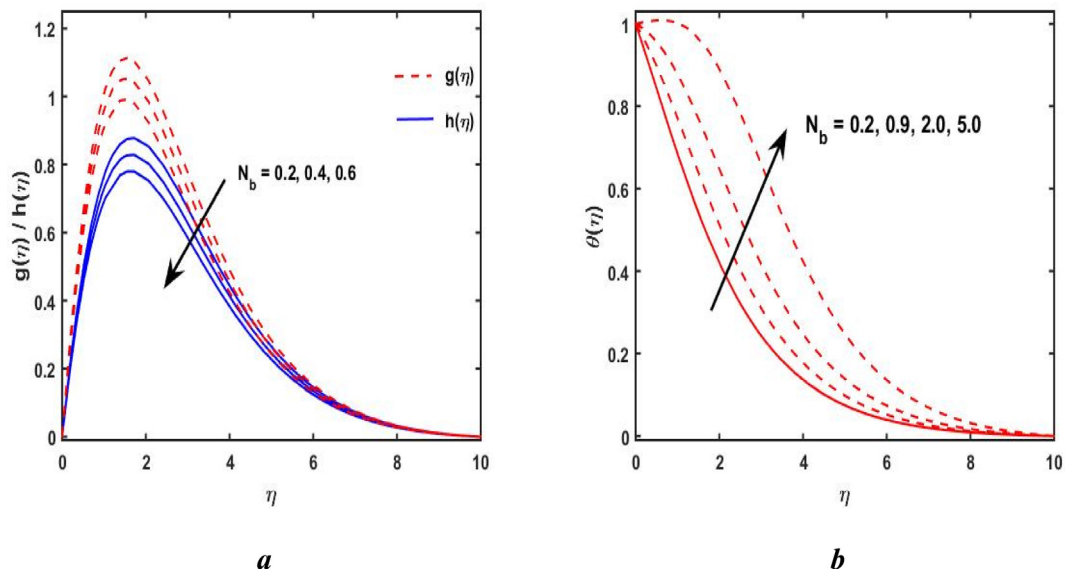


Fig. 4 (a, b): Free convective velocity profile and temperature distribution via  $N_b$ .

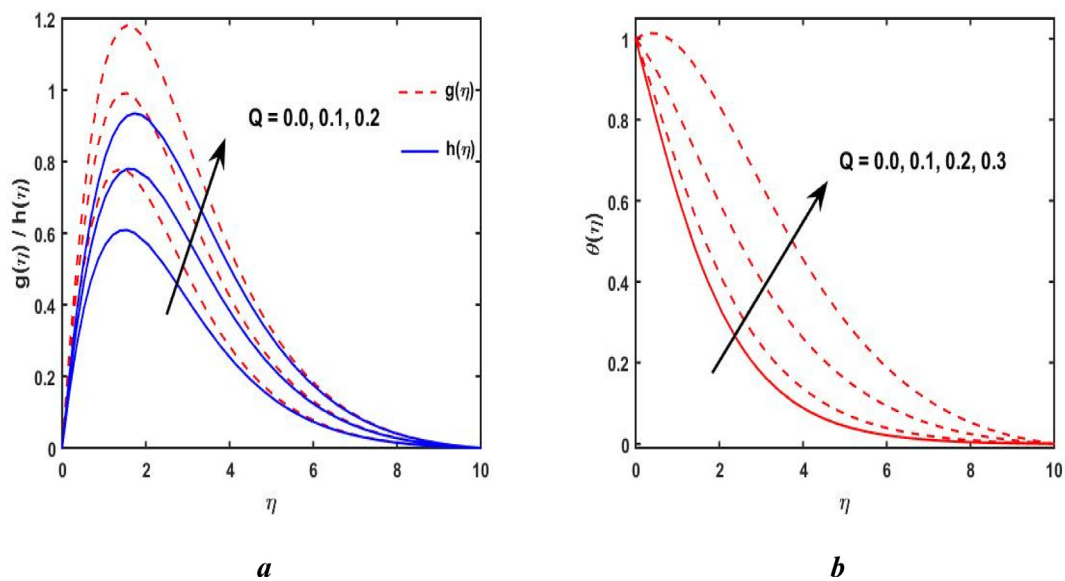


Fig. 5 (a, b): Free convective velocity profile and temperature distribution via  $Q$ .

ity profile is depicted in Fig. 3(a, b). The outcomes prove that both flow and temperature field enhance when thermophoresis parameter  $N_t$  increases. Physically, thermal gradient in the nanofluid boost up the particles motion from high energy level to low energy level due higher trend of  $N_t$  therefore free convective flow and temperature field both enhance. A significant variation in convective velocity profile is seen for higher  $N_t$  because the thermophoresis process is important in free convection flow. The influence of Brownian motion parameter  $N_b$  on flow and thermal field is provided in Fig. 4(a, b). Increase in  $N_b$  boost up the temperature field but declines the free convective velocity field. Physically, the particles random motion increase when  $N_b$  enhances the kinetic energy of

particles transform into heat energy thus, in result the temperature profile elevate. Moreover, this random motion produces the resistive force to fluid motion due to which the convective velocity profile lower down. The heat source in the present flow system provides extra heat from surface of sheet to the fluid which elevates the thermal gradient. Hence, fluid motion due to buoyancy force increase and temperature profile as well. These outcomes are plotted in Fig. 5(a, b). The microorganism concentration due to chemotaxis bioconvection increases with the large magnitude of Peclet number  $Pe$ . Physically, the advective transportation of microorganism due free convective fluid motion is higher than diffusion due the concentration gradient of nanoparticles in the fluid in case of

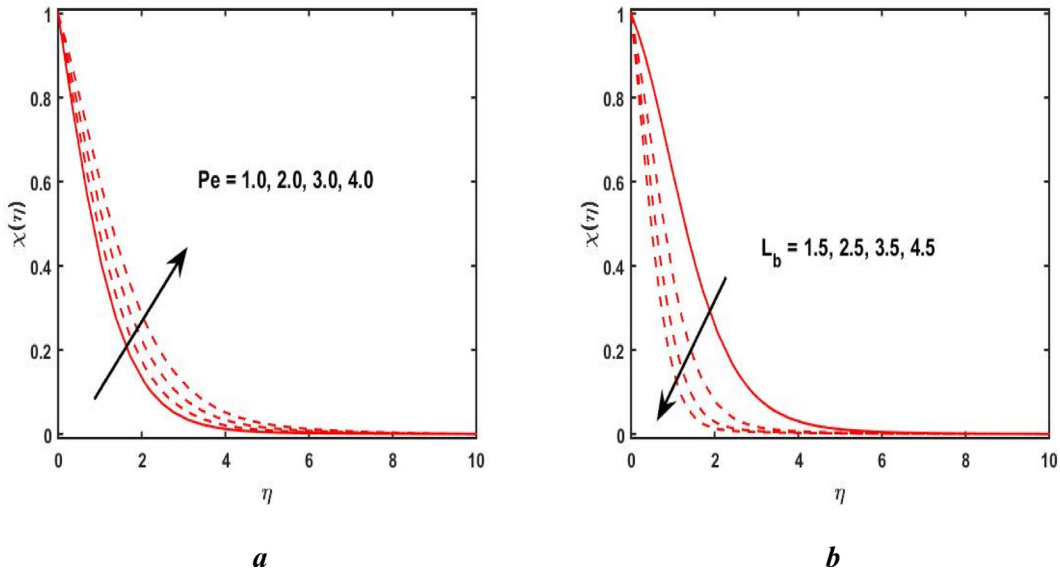


Fig. 6 (a, b): Microorganism profile via  $Pe$  and  $L_b$ .

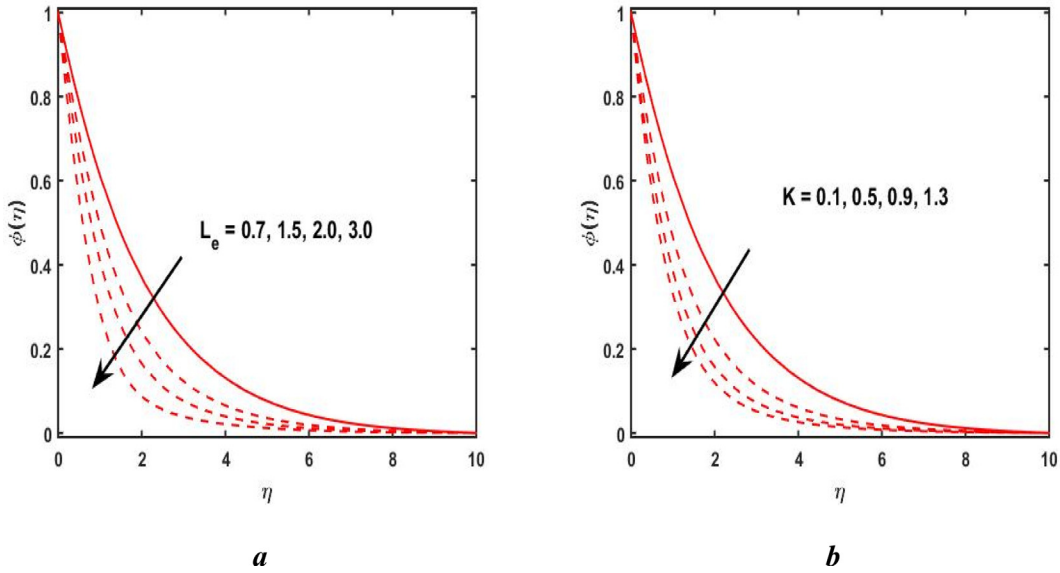


Fig. 7 (a, b): Concentration profile via  $L_e$  and  $K$ .

high  $Pe$  number. Moreover, the swimming velocity of microorganism species is also enhanced as compared to diffusivity. Therefore, the enhancement observed in the concentration of microorganism species (see Fig. 6a). Meanwhile, the bio-convective Lewis number lowers down the concentration profile of microorganism (see Fig. 6b). Because an increase in  $L_b$ , the diffusivity of microorganism species, with a fixed advection rate i.e.  $Pe$  has a fixed value, declines significantly. Fig. 7(a, b) depicts that both the chemical reaction parameter and Lewis number decrease the nanoparticle concentration in the base fluid. Physically, in the presence of a chemical reaction, the flow system's solid particle movement is disturbed. Thus, their concentration decreases in the free convective flow. The forced convective velocity profile is presented in Fig. 8. This profile

is exponentially decayed to free stream. Which means the effect of stretching sheet are zero at infinity. The results for free convective velocity profiles prove that this velocity first increases algebraically in some region near to the sheet and then decays exponentially to the free stream. The peak of the profile depends upon the particular value of the physical parameter. It is seen from Table 1, that the heat transport rate enhances for a stronger estimation of  $Pr$ , while the opposite trend is observed for the greater values of  $Nb$ ,  $Nt$ , and  $Q$ . Furthermore, it is deliberated that the mass transport rate boosts for growing values of  $Pr$ ,  $Nb$ , and  $Q$ , while the reverse trend is seen for  $Nt$ . Table 2 represents the validation of current results with previously published data, which show great harmony among them.

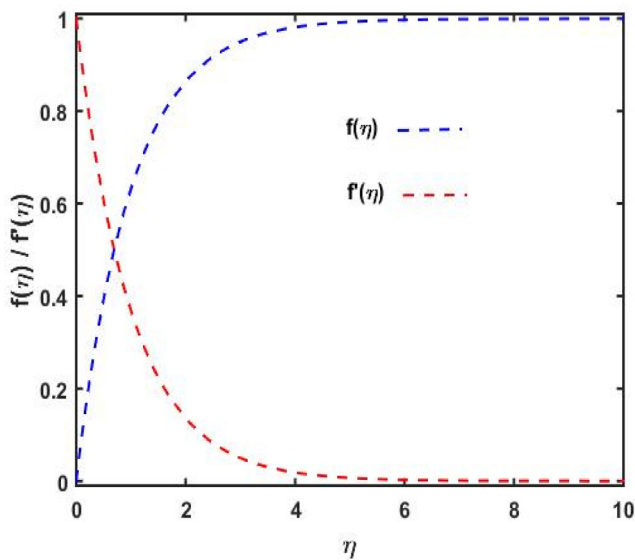


Fig. 8 Forced convection velocity profile.

**Table 1** Table of Nusselt and Sherwood number for various parameters.

$Pr$	$Nb$	$Nt$	$Q$	$-\theta'(0)$	$-\phi'(0)$
0.7				0.2254	0.49033
0.9				0.30086	0.56195
1.0				0.36599	0.62968
	0.2			0.22540	0.49033
	0.4			0.18587	0.52047
	0.6			0.15058	0.52987
		0.0		0.23489	0.53477
		0.3		0.20670	0.42230
		0.6		0.17935	0.37049
			0.0	0.40029	0.42087
			0.1	0.29625	0.46256
			0.2	0.12415	0.52920

**Table 2** Comparison table for the various estimation of Prandtl number along  $g'(0)$ ,  $h'(0)$ , and  $\theta'(0)$ .

$Pr$	$g'(0)$	Ref. (Wang, 1989)	$h'(0)$	Ref. (Wang, 1989)	$\theta'(0)$	Ref. (Wang, 1989)
0.07	-0.0656	-0.06581	5.7460	5.7461	8.690	8.6901
0.2	-0.1691	-0.16920	2.3090	2.3089	3.287	3.2880
0.7	-0.4539	-0.45410	0.9480	0.9475	1.208	1.2090
2.0	-0.9114	-0.91120	0.5265	0.5267	0.6160	0.6163
7.0	-1.8954	-1.89550	0.2829	0.2833	0.3094	0.3095
20.0	-3.3539	-3.35371	0.1704	0.1707	0.1796	0.1798
70.0	-6.4622	-6.46190	0.0927	0.0925	0.0956	0.0955

**4. Concluding remarks**

Free thermal convection and chemotaxis bioconvection in viscous nanofluid flow over a heated vertical stretchable sheet is studied. The thermophoresis and Brownian motion phenomena are incorporated for thermal analysis in nanofluid. This research work is concluded with following key points:

Free convective velocity profile increased in some region near to the sheet and then decay to free steam exponentially. Higher value of Prandtl number decreased the free convective flow. The phenomena of thermophoresis and Brownian motion enhanced the thermal energy transport in viscous fluid. The heat source in the flow boost-up the thermal profile as well free convection velocity. Bioconvection enhanced in case of large Peclet number.

**Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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